Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1 **(a)**	$2n+1 = A(n+1)^2 + Bn^2 \Rightarrow 2n+1 = An^2 + 2An + 1 + Bn^2$	
	$A=1 \ \ B=-1$ or $\dfrac{1}{n^2} - \dfrac{1}{(n+1)^2}$	B1 (1)
(b)	$\sum_{r=5}^{n} \dfrac{2r+1}{r^2(r+1)^2} = \sum_{r=5}^{n} \left(\dfrac{1}{r^2} - \dfrac{1}{(r+1)^2}\right)$ $= \left(\dfrac{1}{5^2} - \cancel{\dfrac{1}{6^2}}\right) + \left(\cancel{\dfrac{1}{6^2}} - \cancel{\dfrac{1}{7^2}}\right) + \ldots \left(\cancel{\dfrac{1}{n^2}} - \dfrac{1}{(n+1)^2}\right)$	M1
	$\sum_{r=5}^{n} \dfrac{2r+1}{r^2(r+1)^2} = \dfrac{1}{5^2} - \dfrac{1}{(n+1)^2}$	A1
	$= \dfrac{n^2+2n+1-25}{25(n+1)^2} = \dfrac{n^2+2n-24}{25(n+1)^2}$	M1A1 (4)
		[5]

	Notes
(a) **B1**	Both values correct with or without working seen, may be in the expression. Ignore incorrect working.
(b) **M1**	Show sufficient terms to demonstrate the cancelling. Require at least one cancelling term seen. Must start at $r = 5$ - M0 if starting at e.g $r = 1$ unless there is a full process to complete the difference method (same condition) **and** apply f(n) −f(4)
A1	Extract the two correct terms, or in the Alt obtains a correct overall expression.
M1	Write the terms with a (non-zero) common denominator with at least numerator correct for their terms. Not dependent - may be scored following M0 if no cancelling terms were shown, but must have had exactly two terms to combine from differences.
A1	Correct answer in the required form or accept correct values stated following an unsimplified form. (Allow as long as correct terms were extracted, even if no cancelling terms were shown.) Note: this means M0A0M1A1 can be scored for answers which show only the last two lines of the scheme with no cancelling process shown. Note: if e.g. r is used in place of n allow full marks if recovered, but A0 if left in terms of r.

Alt (b) for first two marks	$\sum_{r=1}^{n} \dfrac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{n} \left(\dfrac{1}{r^2} - \dfrac{1}{(r+1)^2}\right)$ $= \left(\dfrac{1}{1^2} - \cancel{\dfrac{1}{2^2}}\right) + \left(\cancel{\dfrac{1}{2^2}} - \cancel{\dfrac{1}{3^2}}\right) + \ldots \left(\cancel{\dfrac{1}{n^2}} - \dfrac{1}{(n+1)^2}\right) = 1 - \dfrac{1}{(n+1)^2}$ $\rightarrow \sum_{r=5}^{n} \dfrac{2r+1}{r^2(r+1)^2} = 1 - \dfrac{1}{(n+1)^2} - \left(1 - \dfrac{1}{25}\right) \left(= \dfrac{n^2+2n}{(n+1)^2} - \dfrac{24}{25}\right)$	M1A1

Question Number	Scheme	Marks
2 (a)	E.g. $(x+3)(x-5)=9 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$ OR $(x-5)(x+3)^2-9(x+3)=0 \quad \Rightarrow (x+3)(x-6)(x+4)=0 \Rightarrow x=...$ OR $\dfrac{(x+3)(x-5)-9}{x+3}<0 \Rightarrow x^2-2x-24=0 \Rightarrow x=...$	M1
	CVs: $6,\ -4\ ;\ -3$	A1;B1
	$x<-4, \quad -3<x<6$	dM1A1A1
	OR: $x \in (-3,6) \cup (-\infty,-4)$ or any equivalent notation.	(6)
(b)	$x<6, \quad x \neq -3$ or any equivalent notation.	B1ftB1 (2)
		[8]

	Notes
(a) **M1**	For a correct **algebraic** method to find the intersection points of $y=x-5$ and $y=\dfrac{9}{x+3}$. May set these equal and form a quadratic and solve. May multiply through by $(x+3)^2$ and collect on one side or use any other valid method Eg work from $\dfrac{(x+3)(x+2)-12}{x+3}>0$ Answers only from a calculator score M0. Must reach at least a quadratic or cubic before answers given. Do not be concerned with the equality or inequality for this mark.
A1	For 6, −4 obtained via a valid algebraic method.
B1	for the CV -3 seen anywhere
dM1	Obtaining (any) inequalities using all of their critical values and no other numbers.
A1	For at least one correct interval allowing for … or „ used instead of < and >
A1cso	Both correct ranges and no extras. Use of … or „ scores A0. May be written in set notation, and all work should have been correct so penalise if incorrect inequalities method was used at the start. Accept $x<-4$ and/or $-3<x<6$ with "and" or "or" For candidates who draw a sketch graph and follow with the cvs without any algebra shown only the **B** mark is available. **Those who use some algebra** after their graph may gain marks as earned (possibly all)
(b) **B1ft**	For the "$x<6$" in some form with the possible exception of the CVs from (a). Allow x „ 6 if already penalised in (a). It is essentially for realising all the extra (valid) values less than −3 are solutions while retaining all their given solutions. If only the CVs themselves are excluded allow B1. Follow through their answer to (a).
B1	Fully correct answer. May give as intervals $x<-3, -3<x<6$

Question Number	Scheme	Marks
3	$w=\dfrac{z}{z+4\text{i}}$	
	$w(z+4\text{i})=z\Rightarrow z(1-w)=4\text{i}w$ or $z=\dfrac{4\text{i}w}{1-w}$ oe	M1A1
	$\lvert z\rvert=3$ $\left\lvert\dfrac{4\text{i}w}{1-w}\right\rvert=3$	dM1
	$\lvert 4\text{i}w\rvert=3\lvert 1-w\rvert$	
	$w=u+\text{i}v$ $16\left(u^2+v^2\right)=9\left((1-u)^2+v^2\right)$	ddM1A1
	$16u^2+16v^2=9\left(1-2u+u^2+v^2\right)$	
	$7u^2+7v^2+18u-9=0$	
	$\left(u+\dfrac{9}{7}\right)^2+v^2=\dfrac{144}{49}$	dddM1
	Centre $\left(-\dfrac{9}{7},0\right)$ Radius $\dfrac{12}{7}$	A1A1 (8)
	Notes	
(a) **M1**	re-arrange to $z=\dots$ or an expression $z(\alpha w+\beta)=\gamma w+\delta$	
A1	correct result	
dM1	dep (on first M1) using $\lvert z\rvert=3$ with their previous result	
ddM1	dep (on both previous M marks) use $w=u+\text{i}v$ (or $w=x+\text{i}y$ or any other pair of letters) and attempts the squares of the moduli. The i's must be dealt with correctly, but allow e.g. $3^2\to 3$	
A1	for a correct equation quadratic in u and v after squaring (including squaring coefficients).	
dddM1	dep (on all previous M marks) re-arrange to the completed square form of the equation of a circle (same coeffs for the squared terms) or implied by a correct centre or radius following a correct equation with terms gathered.	
A1	either correct and exact.	
A1	both correct and exact.	
	Note: Allow recovery for the last three A's if all that is incorrect in is the wrong sign in their expression for z, ie $z=\dfrac{-4\text{i}w}{1-w}$	
	If you see alternative methods, e.g. via Apollonian approaches or attempts to use $z=x+\text{i}y$ in the original equation, that you feel are worthy of credit please use Review to consult your team leader.	

Question Number	Scheme	Marks
4	$\dfrac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$	
(a)	$e^{-3\int \tan x\,dx} = e^{-3\ln\sec x} = \sec^{-3} x$ or $\cos^3 x$	M1A1
	$\cos^3 x\dfrac{dy}{dx} - 3y\sin x\cos^2 x = e^{4x}\cos^3 x\sec^3 x$	
	$\dfrac{d}{dx}\left(y\cos^3 x\right) = e^{4x} \Rightarrow y\cos^3 x = \int e^{4x}\,dx$	M1
	$y\cos^3 x = \dfrac{1}{4}e^{4x} \quad (+c)$	M1
	$y = \left(\dfrac{1}{4}e^{4x} + c\right)\sec^3 x$ or $y = \left(\dfrac{1}{4}e^{4x} + c\right)\cos^{-3} x$ oe	A1
		(5)
(b)	$y = 4, \quad x = 0 \quad 4 = \left(\dfrac{1}{4} + c\right)$	
	$c = \dfrac{15}{4}$	M1
	$y = \dfrac{1}{4}\left(e^{4x} + 15\right)\sec^3 x$ or $\dfrac{1}{4}\left(e^{4x} + 15\right)\cos^{-3} x$ oe	A1
		(2)
		[7]

	Notes
(a)	
M1	Attempt the integrating factor, including integration of (-3)tan x; $\ln\cos$ or $\ln\sec$ seen
A1	Correct simplified integrating factor $\sec^{-3} x$ or $\cos^3 x$
M1	Multiply the equation by the integrating factor and integrate the LHS. Look for $y \times \text{their IF} = \int\left(e^{4x}\sec^3 x \times \text{their IF}\right)dx$ (condone missing dx)
M1	Integrate RHS, constant not needed. Must be a function they can integrate and a valid attempt (e.g. allowing coefficient slips).
A1	Correct result in the demanded form, including y =.., constant included
(b)	
M1	Use the given initial conditions to obtain a value for c
A1	Fully correct final answer. Must include y = .. but allow A1 if missing and penalised in (a). May be in the form $y\cos^3 x = \ldots$ or $4y\cos^3 x = \ldots$

Question Number	Scheme	Marks
5.	$\dfrac{d^2y}{dx^2} = -\dfrac{2}{y}\left(\dfrac{dy}{dx}\right)^2 + 2$	
(a)	$\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen	B1
	$\dfrac{d^3y}{dx^3} = -\dfrac{4}{y}\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2} + \dfrac{2}{y^2}\left(\dfrac{dy}{dx}\right)^3$	M1A1A1
		(4)
ALT:	$\left(\dfrac{dy}{dx}\right)^2 \to 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen	B1
	$\underline{\dfrac{dy}{dx}\left(\dfrac{d^2y}{dx^2}\right) + y\dfrac{d^3y}{dx^3}} + 4\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^2y}{dx^2}\right)\underline{-2\dfrac{dy}{dx}} = 0$	M1<u>A1</u>
	$\dfrac{d^3y}{dx^3} = \dfrac{1}{y}\left(-5\dfrac{d^2y}{dx^2} + 2\right)\dfrac{dy}{dx}$	A1 (4)
(b)	At $x = 0$ $\quad \dfrac{d^2y}{dx^2} = \dfrac{1}{2}\left(-2\times(1)^2 + 4\right) = 1$	B1
	$\dfrac{d^3y}{dx^3} = \dfrac{1}{2}(-5\times1+2)\times1 = \dfrac{-3}{2}$	M1
	$(y =)\,2 + x + (1)\dfrac{x^2}{2!} + \left(\dfrac{-3}{2}\right)\dfrac{x^3}{3!} + \ldots$	M1
	$y = 2 + x + \dfrac{1}{2}x^2 - \dfrac{1}{4}x^3 + \ldots$	A1 (4)
		[8]

	Notes
(a)	
B1	$\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ seen in the differentiation
M1	Divide equation by y and differentiate wrt x chain and product rules needed. LHS correct
A1	Either RHS term correct. Need not be simplified.
A1	Both RHS terms correct. Need not be simplified.
ALT	
B1	$\left(\dfrac{dy}{dx}\right)^2 \to 2\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}$ correct differentiation of middle term.
M1	Differentiate before dividing. Product rule must be used.
A1	Correct differentiation of $y\dfrac{d^2y}{dx^2}$ and $-2y$
A1	Rearrange to a correct expression for $\dfrac{d^3y}{dx^3}$ (need not be simplified)

	Notes
(b)	
B1	Correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. May be implied by the term in their expansion.
M1	Use their expression from (a) to obtain a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ (May be implied - you may need to check if their value follows from their expression in (a).)
M1	Taylor's series formed using their values for the derivatives, accept 2! or 2 and 3! or 6
A1	Correct series, must start y =…, or allow f(x) = ... as longs y = f(x) has been defined in the question. Must come from a correct expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

Question Number	Scheme	Marks
6 (a)	$\dfrac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta}=4a\cos\theta+4a\cos^2\theta-4a\sin^2\theta$ or $4a\cos\theta+4a\cos2\theta$ oe (Or allow $\dfrac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta}=-4a\sin\theta-8a\cos\theta\sin\theta$ or $-4a\sin\theta-4a\sin2\theta$)	M1
	E.g. $4a\cos\theta+4a\cos^2\theta-4a\sin^2\theta=0\Rightarrow\cos\theta+\cos^2\theta-(1-\cos^2\theta)=0$	M1
	$2\cos^2\theta+\cos\theta-1=0$ terms in any order	A1
	$(2\cos\theta-1)(\cos\theta+1)=0\Rightarrow\cos\theta=...$	ddM1
	$\left(\cos\theta=\dfrac{1}{2}\Rightarrow\right)\theta=\dfrac{\pi}{3}$ $\quad(\theta=\pi$ need not be seen$)$	A1
	$r=4a\times\dfrac{3}{2}=6a$	A1 (6)
(b)	Area $=\dfrac{1}{2}\int r^2\mathrm{d}\theta=\dfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}16a^2(1+\cos\theta)^2\,\mathrm{d}\theta$	
	$=\dfrac{16a^2}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(1+2\cos\theta+\cos^2\theta)\mathrm{d}\theta$	M1
	$=8a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(1+2\cos\theta+\dfrac{1}{2}(\cos2\theta+1)\right)\mathrm{d}\theta$	M1
	$=8a^2\left[\theta+2\sin\theta+\dfrac{1}{2}\left(\dfrac{1}{2}\sin2\theta+\theta\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$8a^2\left[\dfrac{\pi}{3}+\sqrt{3}+\dfrac{1}{4}\times\dfrac{\sqrt{3}}{2}+\dfrac{\pi}{6}-\left(\dfrac{\pi}{6}+1+\dfrac{1}{4}\times\dfrac{\sqrt{3}}{2}+\dfrac{\pi}{12}\right)\right]$	A1
	$8a^2\left[\dfrac{\pi}{4}+\sqrt{3}-1\right]$	
	Area $R=8a^2\left[\dfrac{\pi}{4}+\sqrt{3}-1\right]-6a^2\left(1+\dfrac{\sqrt{3}}{2}\right)=a^2\left(2\pi+5\sqrt{3}-14\right)$	M1A1 (7)
		[13]
	Notes	
(a) **M1**	Attempt the differentiation of $r\sin\theta$ using product rule or $\sin2\theta=2\sin\theta\cos\theta$ **OR** for this mark only allow differentiation of $r\cos\theta$, inc use of product rule, chain rule or $\cos^2\theta=\dfrac{1}{2}(1\pm\cos2\theta)$ Allow errors in coefficients as long as the form is correct.	
M1	Sets their derivative of $r\sin\theta$ equal to zero and achieves a quadratic expression in $\cos\theta$	
A1	Correct 3 term quadratic in $\cos\theta$ (any multiple, including a)	
ddM1	Dep on both M marks. Solve their quadratic (usual rules) giving one or two roots	
A1	Correct quadratic solved to give $\theta=\dfrac{\pi}{3}$	

	Notes
A1*	Correct r obtained from an intermediate step. Accept as shown in scheme, or $r = 4a\left(1+\cos\frac{\pi}{3}\right) = 6a$ or equivalent in stages. No need to see coordinates together in brackets
(b) **M1** **M1** **dM1** **A1** **A1** **M1** **A1**	Note : first 4 marks of (b) do not require limits. Use of correct area formula, $\frac{1}{2}$ may be seen later, inc squaring the bracket to obtain 3 terms - limits need not be shown. Use double angle formula (formula to be of form $\cos^2\theta = \pm\frac{1}{2}\left(\cos 2\theta \pm 1\right)$) to obtain an integrable function - limits need not be shown, $\frac{1}{2}$ from area formula may be missing, Attempt the integration $\cos\theta \to \pm k\sin\theta$ and $\cos 2\theta \to \pm m\sin 2\theta$ - limits not needed – dep on 2nd M mark but not the first. Note if only two terms arise from squaring allow for $\cos 2\theta \to \pm m\sin 2\theta$ Correct integration – substitution of limits not required (NB Not follow through) Include the $\frac{1}{2}$ and substitute the correct limits in a correct integral. Note may be attempted via integral from 0 to $\frac{\pi}{3}$ minus integral from 0 to $\frac{\pi}{6}$ - but attempts at sector formula for the latter is A0. Attempt the area of the triangle - accept valid attempt even if not subtracted from area. E.g. attempts $\frac{1}{2}OA.OB\sin\frac{\pi}{6}$ Correct final answer in the demanded the form.

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx}=v+x\frac{dv}{dx}$ or $\frac{dv}{dx}=x^{-1}\frac{dy}{dx}-x^{-2}y$ (oe)	M1A1
	$\frac{d^2y}{dx^2}=\frac{dv}{dx}+\frac{dv}{dx}+x\frac{d^2v}{dx^2}$ or $\frac{d^2v}{dx^2}=-x^{-2}\frac{dy}{dx}+x^{-1}\frac{d^2y}{dx^2}+2x^{-3}y-x^{-2}\frac{dy}{dx}$ (oe)	dM1A1
	$3\left(2\frac{dv}{dx}+x\frac{d^2v}{dx^2}\right)-\frac{6}{x}\left(v+x\frac{dv}{dx}\right)+\frac{6xv}{x^2}+3xv=x^2$ (oe in reverse)	ddM1
	$3x\frac{d^2v}{dx^2}+6\frac{dv}{dx}-6\frac{dv}{dx}-\frac{6}{x}v+\frac{6v}{x}+3xv=x^2$	
	$3\frac{d^2v}{dx^2}+3v=x$ *	A1 * (6)
(b)	$3\lambda^2+3=0$ so $\lambda=\pm i$	M1
	$(v=)Ae^{ix}+Be^{-ix}$ or $(v=)C\cos x+D\sin x$	A1
	P.I: Try $(v=)kx\ (+l)$	B1
	$\frac{dv}{dx}=k$ $\frac{d^2v}{dx^2}=0$	
	$3\times0+3(kx\ (+l))=x$	M1
	$k=\frac{1}{3}$ $(l=0)$	
	$v=Ae^{ix}+Be^{-ix}+\frac{1}{3}x$ or $v=C\cos x+D\sin x+\frac{1}{3}x$	A1
	$y=x\left(Ae^{ix}+Be^{-ix}+\frac{1}{3}x\right)$ or $y=x\left(C\cos x+D\sin x+\frac{1}{3}x\right)$	B1ft (6)
		[12]
	Notes	
(a) **M1**	Attempt to find a relevant first derivative from $y=xv$ e.g to get $\frac{dy}{dx}$ or $\frac{dv}{dx}$ - product or quotient rule must be used. Methods via $\frac{d..}{dv}$ would require a chain rule to reach a relevant derivative.	
A1	Correct derivative	
dM1	Attempt to differentiate **their** $\frac{dy}{dx}$ or $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2y}{dx^2}$ or $\frac{d^2v}{dx^2}$ - product rule must be used. Depends on the previous M mark	
A1	Correct expression for $\frac{d^2y}{dx^2}$ or $\frac{d^2v}{dx^2}$	

	Notes
ddM1	Depends on both previous M marks. Substitute **their** $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $y = xv$ in the original equation to obtain a differential equation in v and x. Alternatively substitute **their** $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$ and $v = \frac{y}{x}$ into equation (II) to obtain a differential equation in y and x
A1*	Obtain the **given** equation/original equation with no errors in the working. There must be at least one step shown between the initial substitution and the result
(b)	
M1	Forms correct AE and attempts to solve (accept $3m^2 +3$ (=0) leading to any value(s)).
A1	Correct CF.
B1	Suitable form for PI (ie one that include kx)
M1	Differentiate their PI twice and substitute their derivatives in the equation $3\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 3v = x$
A1	Obtain the correct result (either form). Must be $v =$…
B1ft	Reverse the substitution. Follow through their previous line. Must be $y =$..

Question Number	Scheme	Marks
8 (a)	$\left(\cos\theta+\mathrm{i}\sin\theta\right)^5=\cos5\theta+\mathrm{i}\sin5\theta$	B1
	$=\cos^5\theta+5\cos^4\left(\mathrm{i}\sin\theta\right)+\frac{5\times4}{2!}\cos^3\theta\left(\mathrm{i}\sin\theta\right)^2$ $+\frac{5\times4\times3}{3!}\cos^2\theta\left(\mathrm{i}\sin\theta\right)^3+\frac{5\times4\times3\times2}{4!}\cos\theta\left(\mathrm{i}\sin\theta\right)^4+\left(\mathrm{i}\sin\theta\right)^5$	M1
	$=\cos^5\theta+\underline{5\mathrm{i}\cos^4\theta\sin\theta}-10\cos^3\theta\sin^2\theta\underline{-10\mathrm{i}\cos^2\theta\sin^3\theta}+5\cos\theta\sin^4\theta\underline{+\mathrm{i}\sin^5\theta}$	A1
	$\sin5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$ $=5\left(1-\sin^2\theta\right)^2\sin\theta-10\left(1-\sin^2\theta\right)\sin^3\theta+\sin^5\theta\frac{\mathrm{d}y}{\mathrm{d}x}$ $=5\left(1-2\sin^2\theta+\sin^4\theta\right)\sin\theta-10\left(1-\sin^2\theta\right)\sin^3\theta+\sin^5\theta$	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
	Alternative: Using "$z-\frac{1}{z}$" $z^5-\frac{1}{z^5}=2\mathrm{i}\sin5\theta$ oe	B1
	Binomial expansion of $\left(z-\frac{1}{z}\right)^5$	M1
	$32\sin^5\theta=2\sin5\theta-10\sin3\theta+20\sin\theta$	A1
	Uses double angle formulae etc to obtain $\sin3\theta=3\sin\theta-4\sin^3\theta$ and then use it in their expansion	M1
	$\sin5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ *	A1* (5)
(b)	Let $x=\sin\theta$ $16x^5-20x^3+5x=-\frac{1}{5}\Rightarrow\sin5\theta=...$	M1A1
	$\Rightarrow\theta=\frac{1}{5}\sin^{-1}\left(\pm\frac{1}{5}\right)=38.306$ (or $-2.307, 69.692. 110.306, 141.693, 182.306$)	dM1
	(or in radians −0.0402…0.6685..., 1.216…,1.925…, 2.473…)	
	Two of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1
	All of (awrt) $x=\sin\theta=-0.963,\ -0.555,\ -0.040,\ 0.620,\ 0.938$	A1 (5)
(c)	$\int_0^{\frac{\pi}{4}}\left(4\sin^5\theta-5\sin^3\theta-6\sin\theta\right)\mathrm{d}\theta=\left(\int_0^{\frac{\pi}{4}}\frac{1}{4}\left(\sin5\theta-5\sin\theta\right)-6\sin\theta\right)\mathrm{d}\theta$	M1
	$=\left[\frac{1}{4}\left(-\frac{1}{5}\cos5\theta+5\cos\theta\right)+6\cos\theta\right]_0^{\frac{\pi}{4}}\left(=\left[-\frac{1}{20}\cos5\theta+\frac{29}{4}\cos\theta\right]_0^{\frac{\pi}{4}}\right)$	A1
	$\frac{1}{4}\left[-\frac{1}{5}\cos\frac{5\pi}{4}+5\cos\frac{\pi}{4}-\left(-\frac{1}{5}+5\right)\right]+6\cos\frac{\pi}{4}-6$	
	$=\frac{1}{4}\left[\frac{1}{5}\times\frac{1}{\text{Ö}2}+\frac{5}{\text{Ö}2}-4\frac{4}{5}\right]+\frac{6}{\sqrt{2}}-6$	dM1
	$=\frac{73\sqrt{2}}{20}-\frac{36}{5}$ oe	A1 (4)
		[14]
	Notes	

(a)	
B1	Applies de Moivre correctly. Need not see full statement, but must be correctly applied.
M1	Use binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$ May only show imaginary parts - ignore errors in real parts. Binomial coefficients must be evaluated.
A1	Simplify coefficients to obtain a simplified result with all imaginary terms correct
M1	Equate imaginary parts and obtain an expression for $\sin 5\theta$ in terms of powers of $\sin\theta$ No $\cos\theta$ now
A1*	Correct **given** result obtained from fully correct working with at least one intermediate line wit the $(1-\sin^2\theta)^2$ expanded. Must see both sides of answer (may be split across lines). A0 if equating of imaginary terms is not clearly implied.
(b)	
	Note Answers only with no working score no marks as the "hence" has not been used. But if the first M1A1 gained then dM1 may be implied by a correct answer.
M1	Use substitution $x = \sin\theta$ and attempts to use the result from (a) to obtain a value for $\sin 5\theta$
A1	Correct value for $\sin 5\theta$
dM1	Proceeds to apply arcsin and divide by 5 to obtain at least one value for θ. Note for $\sin 5\theta = \frac{1}{5}$ the values you may see are the negatives of the true answers. FYI: $(5\theta = -11.53...,191.53..., 348.46..., 551.53..., 708.46..., 911.53...)$ (Or in radians $-0.201…$ 3.3428..., 6.0819..., 9.6260..., 12.365..., 15.909…)
A1	Proceeds to take sin and achieve at least 2 different correct values for x or $\sin\theta$
A1	For all 5 values of x or $\sin\theta$ awrt 3 d.p. (allow 0.62 and – 0.04)
(c)	
M1	Use previous work to change the integrand into a function that can be integrated
A1	Correct result after integrating. Any limits shown can be ignored
dM1	Substitute given limits, subtracts and uses exact numerical values for trig functions
A1	Final answer correct (oe provided in the given form)

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